Minimizing the weighted completion time on a single machine with periodic maintenance

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Plan

1. Introduction
2. $1/pm/\sum_{i=1}^{n} W_i C_i$ Problem
3. Resolution
4. Lower bound
5. Computational results
6. Conclusion

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Introduction

Maintenance Activity

Maintenance: Actions necessary for retaining or restoring a piece of equipment, machine, or system to the specified operable condition to achieve its maximum useful life.

Preventive maintenance: Systematic inspection, detection, correction, and prevention of incipient failures, before they become actual or major failures.

Corrective maintenance: Activities undertaken to detect, isolate, and rectify a fault so that the failed equipment, machine, or system can be restored to its normal operable state

Plan

Why/When/Where: Industrial process with single critical machine in the shop. [1]

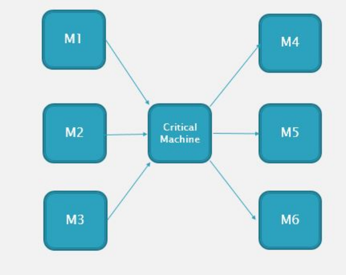

Figure 1 : Example 1

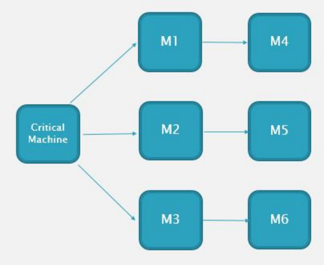

Figure 2 : Example 2

Why/When/Where: Optimize the sum of weighted completion time $\sum_{i=1}^{n} W_i C_i$.

In industry, the $\sum_{i=1}^{n} W_i C_i$ criteria allows to estimate the cost of the stocks.

[1] $\rightarrow$: Flow of semi-finished products

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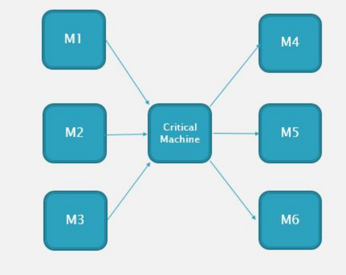

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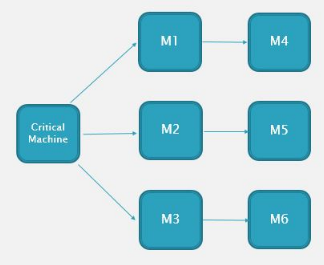

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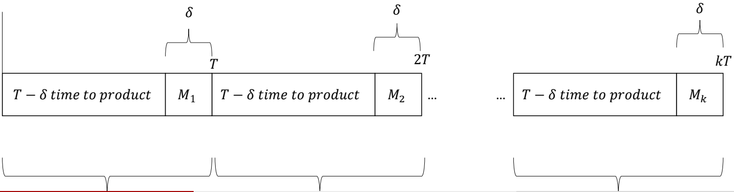
Problem definition

The problem considered can be stated as follows:

- A set $N = \{1, 2, \ldots, n\}$ of n independent jobs
- P_i, S_i, C_i, W_i represent, respectively, the processing time of job i, its starting time, its completion time and its weight.
- Each job, has to be processed without preemption on a single machine that can handle only one job at a time.
- The machine has to undergo a preventive maintenance each T units of time.
- δ is the duration of the maintenance.
- The periodic maintenances defines a batch. We can have at most n batches.



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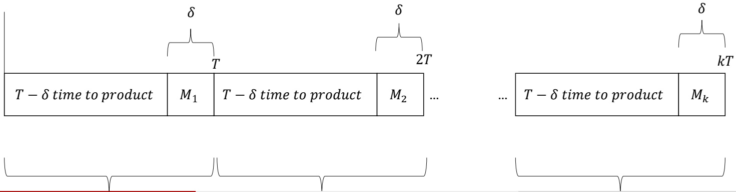

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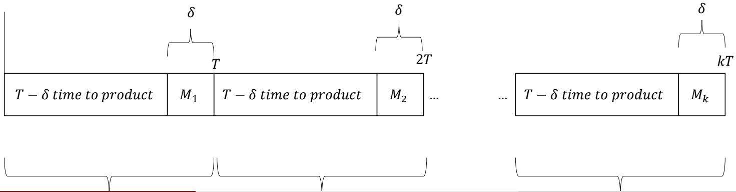

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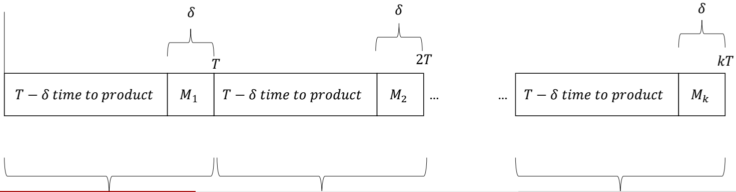

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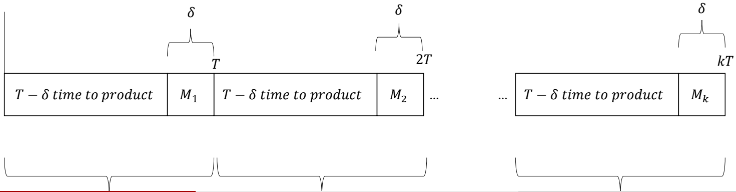

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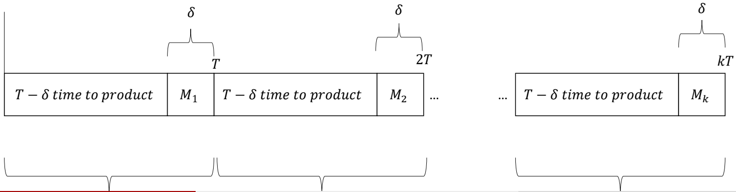

Plan

MIP formulation

We propose a MIP formulation based on precedence variables.

$$x_{ik} = \begin{cases} 1 & \text{if job } i \text{ is scheduled before job } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if job } i \text{ is in batch } j \\ 0 & \text{otherwise} \end{cases}$$

- The set of jobs scheduled between two periodic maintenances defines a batch. We can have at most n batches.
- We define also R as a big positive integer.

Objective function

$$\sum_{i=1}^{n} W_i \,(S_i + P_i)$$

Do not exceed available time per batch

$$\sum_{i=1}^{n} P_i \, y_{ij} \leq T - \delta \quad \forall j \in N$$

Perform all jobs

$$\sum_{j=1}^{n} y_{ij} = 1 \quad \forall i \in N$$

Compute starting times while avoiding overlaps(disjunctive constraints)

$$S_i \geq (j-1)(T)y_{ij} \quad \forall i \in N, \forall j \geq 2$$

$$S_i + p_i \leq S_k + R(1 - x_{ik}) \quad \forall i \in 1..n-1, k \in i+1,..n$$

$$S_k + P_k \leq S_i + Rx_{ik} \quad \forall i \in 1..n-1, k \in i+1,..n$$

Define domain for the variables

$$S_i \geq 0 \quad \forall i \in N$$

$$x_{ik} \in \{0,1\} \quad \forall (i,k) \in N^2$$

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Benchmarks

First benchmark: $\delta = 0$ and fixed T

Why/When/Where:

- Preliminary tests
- Negligible maintenance duration vs T
- Fixed timetable-based maintenance

Second benchmark: $\delta = 2\% T$ and Fixed T

Why/When/Where:

- More realistic tests
- Significant maintenance duration vs T (*Approximately 2 days spent in maintenance every 3 months*)
- Fixed timetable-based maintenance

No breakdowns

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15 instances for $n = 2$ to 20 jobs with $P_i \in [25, 50]$, $W_i \in [1, n]$ and $T = 150$ are randomly generated .

		Time				%CPU			
	Size	average	std,dev	max	min	average	std,dev	max	min
delta = 0	10	2,44	2,55	9,07	0,50	591,20	91,35	724,00	422,00
	11	29,10	34,73	100,52	0,73	684,07	93,65	758,00	464,00
	12	100,50	189,21	731,34	4,42	721,07	34,85	766,00	665,00
	13	351,39	513,26	1200,00	5,79	718,13	29,54	759,00	654,00
	14	638,63	485,58	1200,08	27,86	735,60	28,17	770,00	696,00
	15	1084,72	265,32	1200,06	251,37	717,13	20,94	767,00	701,00
	16	999,35	403,90	1200,07	73,33	717,73	22,66	766,00	667,00
	17	1200,06	0,01	1200,07	1200,05	708,20	18,36	725,00	650,00
	18	1200,04	0,01	1200,06	1200,00	713,93	12,34	736,00	695,00
	19	1200,04	0,01	1200,06	1200,00	710,80	16,85	731,00	672,00
	20	1200,04	0,01	1200,08	1200,02	698,60	16,62	721,00	653,00
delta = 2%T	10	1,97	1,86	6,64	0,56	582,40	77,77	704,00	460,00
	11	19,23	23,65	87,99	0,66	673,33	100,69	755,00	465,00
	12	58,33	84,15	251,78	3,85	718,73	37,23	768,00	653,00
	13	338,45	520,01	1200,08	4,75	713,73	28,60	754,00	642,00
	14	596,76	521,72	1200,09	15,47	731,27	25,22	765,00	691,00
	15	977,32	382,88	1200,07	123,85	716,80	25,99	764,00	670,00
	16	1008,31	400,81	1200,07	53,36	709,20	26,16	761,00	648,00
	17	1141,74	161,61	1200,06	589,16	712,93	15,62	740,00	682,00
	18	1200,05	0,01	1200,10	1200,04	707,87	15,86	733,00	671,00
	19	1200,05	0,02	1200,09	1200,00	708,07	14,31	729,00	680,00
	20	1200,04	0,01	1200,07	1200,02	699,87	13,53	723,00	682,00

Figure 3 : Computational results of the MIP for some instances

Some properties

Property

The jobs in each batch are scheduled by Smith's rule. (smith, 1956).

Property

If batch B_j is before batch B_{j+1} then in the optimal solution we have

$$\sum_{i \in B_j} w_i \geq \sum_{k \in B_{j+1}} w_k$$

Heuristics

The MIP model takes more than 20 minutes to solve optimally instances with more than 12 jobs.

We present here 9 combination of heuristics designed to solve the big instances.

Step 1: Form a sequencing priority list. Rank the job in :

- Decreasing order of processing time P_i.
- Increasing order of processing time P_i.
- Decreasing order of $\frac{P_i}{w_i}$

Step 2: Determine a minimum number of batches. Using:
First fit, Best fit or Next fit strategy.

Step 3: Compute W_j, the sum of the weights of all the jobs T_i who belong to the same Batch j

Step 4: Rank the batches in decreasing order of W_j

Step 5: Rank the jobs in increasing order of $\frac{P_i}{w_i}$ in each batch.

Plan

Proposition

$LB1 = \sum_{j=1}^{n} P_j \sum_{i=j}^{n} W_i$ is a valid lower bound.

Proof.

The WSPT rule can obtain an optimal solution of this relaxed problem[**?**]. Let us consider f^* its optimal objective value.

We suppose that the n jobs are sorted in increasing order of the WSPT rule.

$L = \{P_1, P_2, ..., P_n\}$. So we have:

$f^* = \sum_{i=1}^{n} W_i C_i$

$= W_1 C_1 + W_2 C_2 + ... + W_n C_n$

$= W_1 P_1 + W_2 (P_1 + P_2) + W_3 (P_1 + P_2 + P_3) ... + W_n (P_1 + P_2 + P_3 + ... P_n).$

$= P_1 \sum_{i=1}^{n} W_i + P_2 \sum_{i=2}^{n} W_i + P_3 \sum_{i=3}^{n} W_i + ... + P_{n-1} \sum_{i=n-1}^{n} W_i + P_n W_n x =$
$\sum_{j=1}^{n} P_i \sum_{i=j}^{n} W_i.$ □

Plan

Comparison MIP/Heuristic

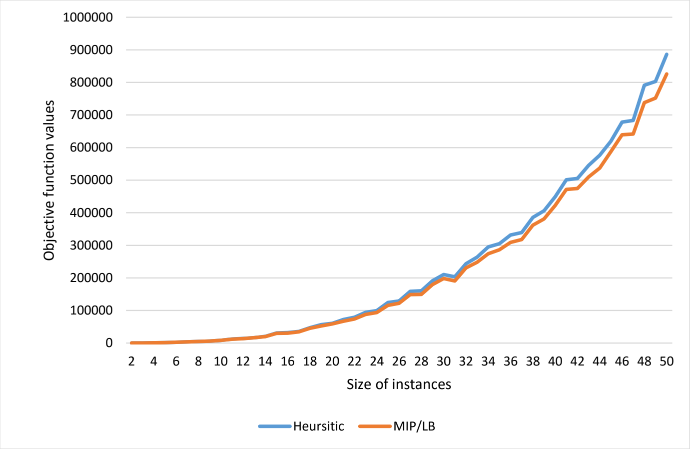

Figure 4 : Comparison of results-heuristics vs MIP

		WCT	Gap % on objective function	Gap %
Size	Seed	Delta=0	WSPTBF	L.B.
10	289	8211	0,60	-5,07
10	1517	7110	1,24	-4,42
10	5601	9518	0,28	-5,06
10	6174	7348	0,00	-3,18
10	10025	9137	0,00	-1,09
10	11606	8442	0,00	-3,51
10	11643	7024	2,51	-3,47
10	13617	6092	0,69	-3,94
10	15093	9015	7,04	-2,42
10	17396	7878	0,00	-3,26
10	20460	6618	2,13	-7,03
10	22374	9511	6,70	-4,36
10	30160	5980	2,39	-2,04
10	30778	9616	0,67	-5,37
10	31882	9452	1,49	-1,82
		Average	1,72	-3,74

Figure 5 : Computational results for 15 instances with $n = 10$

Plan

Conclusion

In this work:

- We investigate a single machine with periodic preventive maintenance
- The objective of minimizing the weighted completion time is adressed and explained
- The mathematical model is efficient on instances with size ≤ 20
- Several heurictics are compared and applicable on large instances
- A lower bound is also proposed.

Perspective

Potential direction of our future research would be :

- Add the constraint of unpredicted situation (failure, order cancellation,etc)
- Consider more than one machine in the shop
- Resolve the problem with metaheuristics

Thank you for your attention