Single machine scheduling and outbound delivery problems

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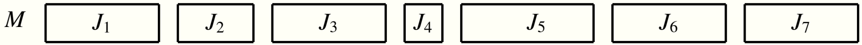
Plan

Context

Coordinating production and delivery schedules.



Given

- A set of n jobs $\{J_1, J_2, \ldots, J_n\}$
- A single machine M
- A processing time p_j for each job J_j
- a location j for each job J_j

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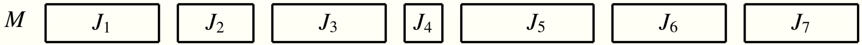

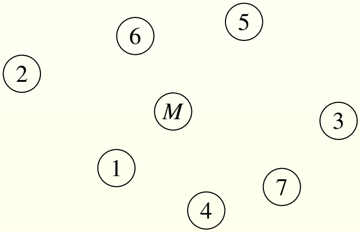

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Each job must be delivered and belong to exactly one route.

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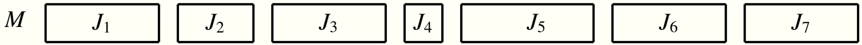
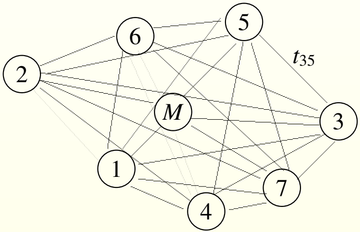




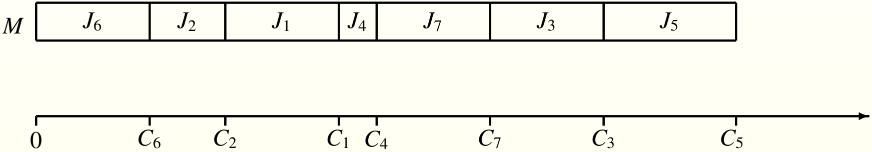
Given

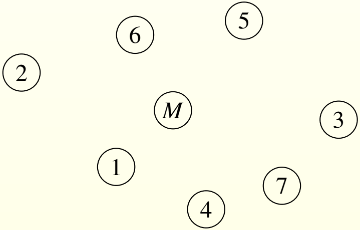
- a single vehicle with a capacity c
- a size of s_j for each job J_j
- the traveling time between i and j is t_{ij}

The set of jobs delivered during a single round trip constitutes a batch

Problem definition

Coordinating production and delivery schedules.





The problem is then to :

- determine the scheduling sequence

where C_j is the completion time of J_j

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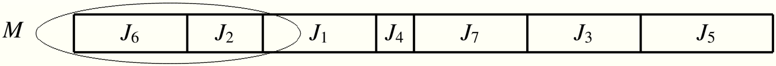
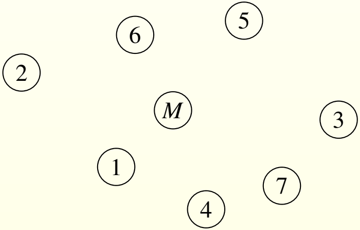


The problem is then to :

- determine the scheduling sequence
- cluster jobs into batches



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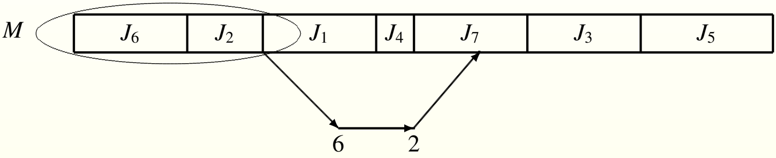
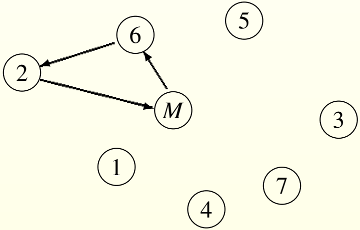


The problem is then to :

- determine the scheduling sequence
- cluster jobs into batches
- determine the best route for each batch



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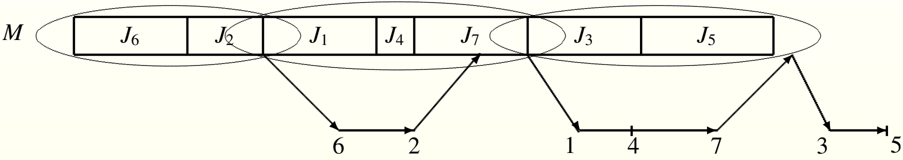

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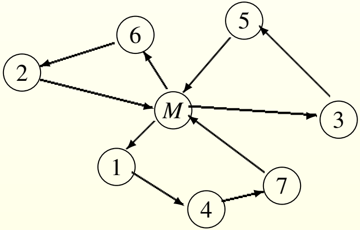

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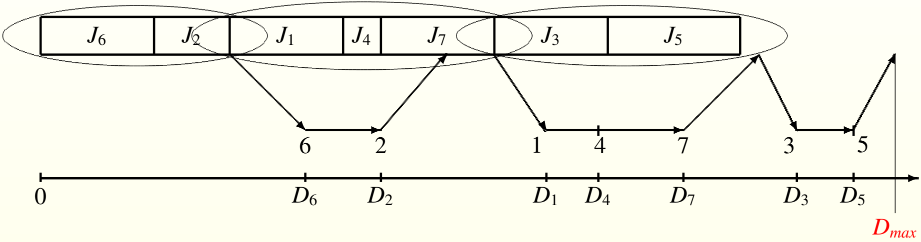

Objective :

- minimize D_{max}

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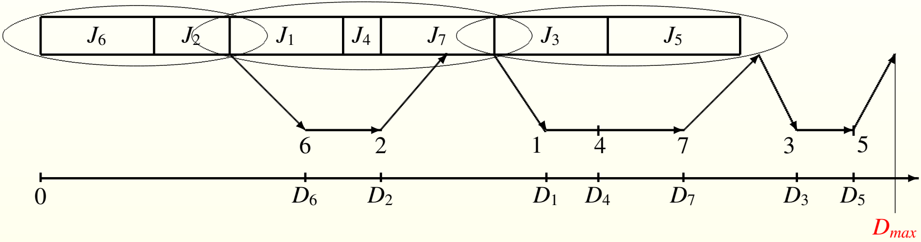


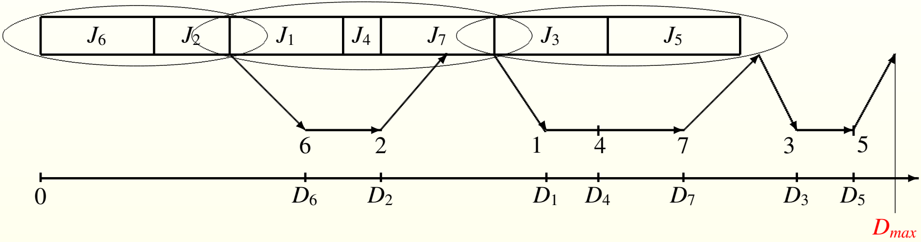
Objective :

- minimize D_{max}

The problem is denoted : $1 \rightarrow D, k \geq 1|v = 1, c|D_{max}$

Property

Coordinating production and delivery schedules.

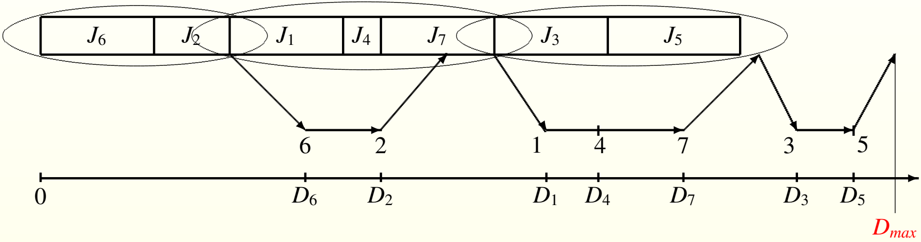


There exists an optimal solution that satisfies the following conditions :

- Jobs are processed on the machine without idle time
- Production sequence and routing sequence are the same

Complexity

Coordinating production and delivery schedules.



Problem $1 \to D, k \geq 1|v = 1, c|D_{max}$ is NP-hard in the strong sense even for the one customer case. [Chang and Lee., 2004]

Plan

A set covering formulation for $1 \rightarrow D, k \geq 1|v = 1, c|D_{max}$

- $\beta = \{b_1, \ldots, b_{|\beta|}\}$ a set of feasible batches for which the positions are fixed, and for each batch one have :
  - $P_{b,1} = \sum_{j \in b} p_j$ the duration of the batch b on the machine
  - $P_{b,2}$ the duration of the round trip that delivers the batch b
  - $a_{i,b} = 1$ if the job J_i is in the batch b
- $x_{b,k} \in \{0, 1\}$ indicates if the batch b at the position k is selected

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$$minimize\ Dmax$$

$$Dmax \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \qquad \forall l \in \{1, ..., n\}$$

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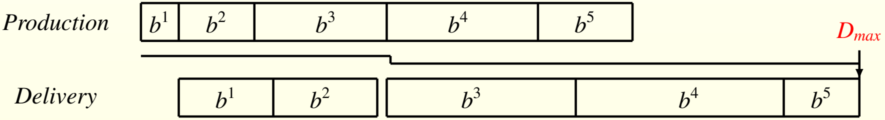

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$$\sum_{b \in \beta} (a_{i,b} \cdot \sum_{k=1}^{n} x_{b,k}) = 1, \qquad \forall i \in \{1, ..., n\}$$

$$\sum_{b \in \beta} x_{b,k} \geq \sum_{b \in \beta} x_{b,k+1}, \qquad \forall k \in \{1, ..., n-1\}$$

Minimum number of round trips

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A lower bound δ for the number of selected positions using FFD algorithm

$$\sum_{b \in \beta} x_{b,k} = 1, \qquad \forall k \in \{1, ..., \delta\}$$

First Fit Decreasing is a $3/2-$approximation for Bin Packing.

Final model

$$minimize\ Dmax \tag{1}$$

$$Dmax \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \qquad \forall l \in \{1, ..., n\} \tag{2}$$

$$\sum_{b \in \beta} x_{b,k} \leq 1, \qquad \forall k \in \{1, ..., n\} \tag{3}$$

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The set of feasible batches β represent too many variables
$\rightarrow$ resolved by column generation

Principle of the column generation

The process works as follows :

- the master problem is solved
  - problem restricted to a sub set of variables (basic and nonbasic)
  - integrity constraints are relaxed
- the subproblem is solved
  - find new nonbasic variable that can improve the solution

The process is repeated until no negative reduced cost variables are identified

The pricing problem (Definition of the subproblem)

- The sub-problem research an element of β such that :

$\rightarrow$ Negative reduced $\bar{c}_{b,k}$ cost for a new column $x_{b,k}$

$$\bar{c}_{b,k} \leq 0 \quad \Leftrightarrow \sum_{i=1}^{n} \frac{p_i \cdot \sum_{l=k}^{n} \alpha_l - \gamma_i}{\sum_{l=1}^{k} \alpha_l} \cdot a_{i,b} + P_{b,2} \leq \frac{\beta_k - \sigma_{k-1} + \sigma_k + \xi_k}{\sum_{l=1}^{k} \alpha_l}$$

where α_l, β_k and γ_i the dual values associated to the constraints.

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$\Rightarrow$ Elementary shortest path problem with resource constraints (ESPPRC) [Lozano et al., 2015]
where $t_{i,j} + A_i$ is the new distance for an arc (i,j)

Instance generation procedure

- processing times $p_i \in [1, 50]$
- t_{ij} set as the distance between of job points (X_i, Y_i) with $X_i, Y_i \in [1, 50]$
- $s_j = 1$ for each job J_j
- capacity $c = 10$

Generation of 5 instances for each $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$

Results

n	$Time(sec)LB$	$Time(sec)UB$	$\#columns$	$\%GAP(Avg)$
10	0.1	0.1	101	5.87
20	1.6	0.3	330	10.83
30	11.2	14.8	1000	7.40
40	62.7	1531.6	1918	7.85
50	769.3	2923.4	4323	7.45
60	786.5	2643.0	3230	6.12
70	3414.5	9258.3	18534	11.01
80	1322.5	5187.6	4273	15.85
90	1794.6	6528.2	6244	6.58

Conclusion

- integrated production and routing problems and particular cases
- a first version of the column generation method
- a good lower bounds on the optimal value

Work in progress

- a branch and price algorithm

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Thank you for your attention

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Alessandro Agnetis, Mohamed Ali Aloulou, and Liang-Liang Fu, (2014). Coordination of production and interstage batch delivery with outsourced distribution, *European Journal of Operational Research*, 238 (1), pp. 130–142.

Armstrong, Ronald and Gao, Su and Lei, Lei, (2008). A zero-inventory production and distribution problem with a fixed customer sequence, *Annals of Operations Research*, 159 (1), pp. 395–414.

A.Cheref and A.Agnetis and C.Artigues and J-C.Billaut, (2015). Fixed sequence integrated production and routing problems, *12th Workshop on Models and Algorithms for Planning and Scheduling Problems (MAPSP 2015)*.

C. Lenté and Y. Kergosien, (2014). Problème de livraison a séquence fixée, *10ème conférence internationale de modélisation, optimisation et simulation (MOSIM'14), Nancy*.

Yung-Chia Chang and Chung-Yee Lee, (2004). Machine scheduling with job delivery coordination, *European Journal of Operational Research*, 158 (2), pp. 470–487.

Zhi-Long Chen, (2010). Integrated Production and Outbound Distribution Scheduling : Review and Extensions, *Operations Research*, 58 (1), pp. 130–148.

Bo Chen and Chung-Yee Lee, (2008). Logistics scheduling with batching and transportation, *European Journal of Operational Research*, 189 (3), pp. 871–876.

Michael R. Garey and Robert E. Tarjan and Gordon T. Wilfong. (1988). One-Processor Scheduling with Symmetric Earliness and Tardiness Penalties, *Mathematics of Operations Research*, 13 (2), pp. 330–348.

Lee, Chung-Yee and Chen, Zhi-Long, (2001). Machine scheduling with transportation considerations, *Journal of Scheduling*, 4 (1), pp. 3–24.

Chung-Lun Li and George Vairaktarakis and Chung-Yee Lee (2005). Machine scheduling with deliveries to multiple customer locations, *European Journal of Operational Research*, 164 (1), pp. 39–51.

Li, Chung-Lun and Ou, Jinwen, (2005). Machine scheduling with pickup and delivery, *Naval Research Logistics*, 52 (7), pp. 617–630.

Ik Sun Lee, (2015). A coordinated scheduling of production-and-delivery under dynamic delivery cost environments, *Computers & Industrial Engineering*, 81 (0), pp. 22–35.

Viergutz, Christian and Knust, Sigrid, (2014). Integrated production and distribution scheduling with lifespan constraints, *Annals of Operations Research*, 213 (1), pp. 293–318.

Feillet, D and Dejax, P and Gendreau, M and Gueguen, C An exact algorithm for the elementary shortest path problem with resource constraints : Application to some vehicle routing problems *Networks*, 44 (2004), pp. 216229