Single machine scheduling and outbound delivery problems

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Plan

1. The problem
2. A column generation scheme
3. Results
4. Conclusion
The problem
A column generation scheme
results
Conclusion

Context

Coordinating production and delivery schedules.

\[ M \begin{array}{ccccccc}
 J_1 & J_2 & J_3 & J_4 & J_5 & J_6 & J_7 \\
\end{array} \]

Given

- A set of \( n \) jobs \( \{J_1, J_2, \ldots, J_n\} \)
- A single machine \( M \)
- A processing time \( p_j \) for each job \( J_j \)
- A location \( j \) for each job \( J_j \)
Context

Coordinating production and delivery schedules.

Given
- a set of $n$ jobs $\{J_1, J_2, \ldots, J_n\}$
- a single machine $M$
- a processing time $p_j$ for each job $J_j$
- a location $j$ for each job $J_j$

Each job must be delivered and belong to exactly one route.
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Given
- a single vehicle with a capacity $c$
- a size of $s_j$ for each job $J_j$
- the traveling time between $i$ and $j$ is $t_{ij}$

The set of jobs delivered during a single round trip constitutes a batch
The problem is then to:

- determine the scheduling sequence

where $C_j$ is the completion time of $J_j$
The problem is then to:

- determine the scheduling sequence
- cluster jobs into batches
Problem definition

Coordinating production and delivery schedules.

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- determine the best route for each batch
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Objective:
- minimize $D_{max}$
Problem definition

The problem is denoted: $1 \rightarrow D, k \geq 1 | v = 1, c | D_{\text{max}}$

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Coordinating production and delivery schedules.

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Coordinating production and delivery schedules.

There exists an optimal solution that satisfies the following conditions:

- Jobs are processed on the machine without idle time
- Production sequence and routing sequence are the same
Coordinating production and delivery schedules.

Problem $1 \rightarrow D, k \geq 1 \mid v = 1, c \mid D_{max}$ is NP-hard in the strong sense even for the one customer case. [Chang and Lee., 2004]
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A set covering formulation for $1 \rightarrow D, k \geq 1|v = 1, c|D_{\text{max}}$

- $\beta = \{b_1, \ldots, b_{|\beta|}\}$ a set of feasible batches for which the positions are fixed, and for each batch one have:
  - $P_{b,1} = \sum_{j \in b} p_j$ the duration of the batch $b$ on the machine
  - $P_{b,2}$ the duration of the round trip that delivers the batch $b$
  - $a_{i,b} = 1$ if the job $J_i$ is in the batch $b$
- $x_{b,k} \in \{0, 1\}$ indicates if the batch $b$ at the position $k$ is selected
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\[
\text{minimize } D_{max}
\]

\[
D_{max} \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \quad \forall l \in \{1, \ldots, n\}
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\[
\begin{align*}
\text{minimize } D_{\text{max}} \\
D_{\text{max}} &\geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, & \forall l \in \{1, \ldots, n\} \\
\sum_{b \in \beta} x_{b,k} &\leq 1, & \forall k \in \{1, \ldots, n\} \\
\sum_{b \in \beta} (a_{i,b} \cdot \sum_{k=1}^{n} x_{b,k}) &\geq 1, & \forall i \in \{1, \ldots, n\} \\
\sum_{b \in \beta} x_{b,k} &\geq \sum_{b \in \beta} x_{b,k+1}, & \forall k \in \{1, \ldots, n-1\}
\end{align*}
\]
Minimum number of round trips

\[ \text{minimize } D_{\text{max}} \]

\[ D_{\text{max}} \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \quad \forall l \in \{1, ..., n\} \]

\[ \sum_{b \in \beta} x_{b,k} \leq 1, \quad \forall k \in \{1, ..., n\} \]

\[ \sum_{b \in \beta} (a_{i,b} \cdot \sum_{k=1}^{n} x_{b,k}) = 1, \quad \forall i \in \{1, ..., n\} \]

\[ \sum_{b \in \beta} x_{b,k} \geq \sum_{b \in \beta} x_{b,k+1}, \quad \forall k \in \{1, ..., n - 1\} \]

A lower bound \( \delta \) for the number of selected positions using FFD algorithm

\[ \sum_{b \in \beta} x_{b,k} = 1, \quad \forall k \in \{1, ..., \delta\} \]

First Fit Decreasing is a \( 3/2 \)—approximation for Bin Packing.
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Final model

\[ \text{minimize } D_{\text{max}} \]  \hspace{1cm} (1)

\[ D_{\text{max}} \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \quad \forall l \in \{1, \ldots, n\} \]  \hspace{1cm} (2)

\[ \sum_{b \in \beta} x_{b,k} \leq 1, \quad \forall k \in \{1, \ldots, n\} \]  \hspace{1cm} (3)

\[ \sum_{b \in \beta} (a_{i,b} \cdot \sum_{k=1}^{n} x_{b,k}) = 1, \quad \forall i \in \{1, \ldots, n\} \]  \hspace{1cm} (4)

\[ \sum_{b \in \beta} x_{b,k} \geq \sum_{b \in \beta} x_{b,k+1}, \quad \forall k \in \{1, \ldots, n-1\} \]  \hspace{1cm} (5)

\[ \sum_{b \in \beta} x_{b,k} = 1, \quad \forall k \in \{1, \ldots, \delta\} \]  \hspace{1cm} (6)
minimize $D_{max}$  

$$D_{max} \geq \sum_{k=1}^{l} \sum_{b \in \beta} P_{b,1} \cdot x_{b,k} + \sum_{k=l}^{n} \sum_{b \in \beta} P_{b,2} \cdot x_{b,k}, \quad \forall l \in \{1, \ldots, n\}$$  

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$$\sum_{b \in \beta} x_{b,k} = 1, \quad \forall k \in \{1, \ldots, \delta\}$$  

The set of feasible batches $\beta$ represent too many variables  
→ resolved by column generation
Principle of the column generation

The process works as follows:

- **the master problem is solved**
  - problem restricted to a sub set of variables (basic and nonbasic)
  - integrity constraints are relaxed

- **the subproblem is solved**
  - find new nonbasic variable that can improve the solution

The process is repeated until no negative reduced cost variables are identified
The pricing problem (Definition of the subproblem)

- The sub-problem research an element of $\beta$ such that:
  - Negative reduced $\bar{c}_{b,k}$ cost for a new column $x_{b,k}$

\[
\bar{c}_{b,k} \leq 0 \iff \sum_{i=1}^{n} \frac{p_i \cdot \sum_{l=k}^{n} \alpha_l - \gamma_i}{\sum_{l=1}^{k} \alpha_l} \cdot a_{i,b} + P_{b,2} \leq \frac{\beta_k - \sigma_{k-1} + \sigma_k + \xi_k}{\sum_{l=1}^{k} \alpha_l}
\]

where $\alpha_l$, $\beta_k$ and $\gamma_i$ the dual values associated to the constraints.
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\]

⇒ Elementary shortest path problem with resource constraints (ESPPRC) [Lozano et al., 2015]
where $t_{i,j} + A_{i}$ is the new distance for an arc $(i,j)$
Instance generation procedure

- processing times $p_i \in [1, 50]$
- $t_{ij}$ set as the distance between job points $(X_i, Y_i)$ with $X_i, Y_i \in [1, 50]$
- $s_j = 1$ for each job $J_j$
- capacity $c = 10$

Generation of 5 instances for each $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$
## Results

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\text{Time (sec) LB}$</th>
<th>$\text{Time (sec) UB}$</th>
<th>#columns</th>
<th>$%\text{GAP (Avg)}$</th>
</tr>
</thead>
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<td>0.1</td>
<td>101</td>
<td>5.87</td>
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<tr>
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<td>10.83</td>
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<td>14.8</td>
<td>1000</td>
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<tr>
<td>40</td>
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<td>1531.6</td>
<td>1918</td>
<td>7.85</td>
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<td>18534</td>
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<tr>
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<td>5187.6</td>
<td>4273</td>
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<tr>
<td>90</td>
<td>1794.6</td>
<td>6528.2</td>
<td>6244</td>
<td>6.58</td>
</tr>
</tbody>
</table>
Conclusion

- integrated production and routing problems and particular cases
- a first version of the column generation method
- a good lower bounds on the optimal value

Work in progress

- a branch and price algorithm
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- a first version of the column generation method
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Thank you for your attention

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